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Operator product expansion for pure spinor superstring on $AdS_5 \times S^5$

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Abstract: The tree-level operator product expansion coefficients of the matter currents are calculated in the pure spinor formalism for type IIB superstring in the $AdS_5 \times S^5$ background.

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1. Introduction and summary

Type IIB superstring in an $AdS_5 \times S^5$ background is conjectured to be dual to $\mathcal{N} = 4$ super Yang-Mills theory in $D = 4$ dimensions. To fully exploit the duality one would need to solve the world-sheet sigma model with the $AdS_5 \times S^5$ target-space at the quantum level, and in particular to understand its operator algebra. As a first step toward this ambitious goal, we analyze in this paper the tree-level OPE of matter currents on the world-sheet.

The type IIB superstring in the AdS space with Ramond-Ramond flux can be formulated as a sigma model with the target space which is the supermanifold $\frac{PSU(2,2|4)}{SO(4,1)\times SO(5)}$. The action in the Green-Schwarz (GS) formalism is known [1]. The pure spinor (PS) version was proposed in [2]. In both these approaches the target-space supersymmetry is manifest. However in the GS formulation where the world-sheet action is classically κ-invariant, the covariant quantization of the sigma model is rather complicated due to non-linearities and because gauge-fixing the κ-symmetry leads to fermionic second-class constraints. In the PS formalism proposed by Berkovits, the main ingredients are the commuting left and right-moving space-time spinors λ^{α} and $\hat{\lambda}^{\hat{\alpha}}$, which play the role of ghost variables and satisfy the *pure spinor* constraint:

$$\lambda\gamma^{\underline{a}}\lambda = \hat{\lambda}\gamma^{\underline{a}}\hat{\lambda} = 0. \tag{1.1}$$

The world-sheet superstring action is classically BRST-invariant in the Berkovits formulation: due to the presence of a kinetic term for the fermionic currents the κ-symmetry of the GS superstring action is replaced by a BRST-like symmetry whose charges are constructed from fermionic constraints and pure spinors. In both these formalisms an infinite set of non-local classically conserved charges has been found, which highly suggest the integrability of the model [6, 5]. At the classical level these non-local charges have been shown to be κ-invariant in the GS formalism and BRST-invariant in the PS formalism [7]. By cohomology arguments it was proved that the BRST invariance of the PS superstring action survive at the quantum level [12]. Moreover the superstring action was explicitly proved to be one-loop conformally invariant in [3]. The (classical) current algebra in the hamiltonian formalism was analyzed in [14]. Here we will compute the operator product expansion (OPE) of the matter currents.

In section 2 the effective action for the fluctuations fields is derived. The results for the OPEs is presented in section 3. Our notation is summarized in the appendices, where we also give some calculational details.

2. The action

In the pure spinor formalism the sigma model action describing an $AdS_5 \times S_5$ background with Ramond-Ramond flux is [2–5, 12]

$$\begin{aligned} S_{AdS} = & \frac{1}{\alpha^2}\int d^2z\{< J_2, \overline{J}_2 > + \frac{3}{2} < J_3, \overline{J}_1 > + \frac{1}{2} < \overline{J}_3, J_1 >\} + \\ & + \frac{1}{\alpha^2}\int d^2z\{N_{\underline{cd}}\overline{J}^{[\underline{cd}]} + \hat{N}_{\underline{cd}}J^{[\underline{cd}]} + \frac{1}{2}N_{\underline{cd}}\hat{N}^{\underline{cd}}\} + \frac{1}{\alpha^2}(S_\lambda + S_{\hat{\lambda}}), \end{aligned} \tag{2.1}$$

where $<,>$ is the bilinear form expressed in terms of the super-trace, $J = J^A T_A$, $\overline{J} = \overline{J}^A T_A$, with T_A the generators of the super-algebra and $J^A = (g^{-1}\partial g)^A$ and $\overline{J}^A = (g^{-1}\overline{\partial} g)^A$ are the left invariant (super) currents constructed from $g(x, \theta, \hat{\theta})$ which are elements of the super-coset $\frac{PSU(2,2|4)}{SO(4,1)\times SO(5)}$ and $(x, \theta, \hat{\theta})$ parameterize the $D = 10$, $N = 2$ superspace. $N_{\underline{cd}} = \frac{1}{2}\omega\gamma_{\underline{cd}}\lambda$ and $\hat{N}_{\underline{cd}} = \frac{1}{2}\hat{\omega}\gamma_{\underline{cd}}\hat{\lambda}$ are the $SO(4,1) \times SO(5)$ components of the Lorentz currents for the pure spinor ghosts λ^α and $\hat{\lambda}^{\hat{\alpha}}$ and their conjugate momenta ω_α and $\hat{\omega}_{\hat{\alpha}}$, respectively. S_λ and $S_{\hat{\lambda}}$ are the free field actions for the pure spinors in the flat background. The action is manifestly invariant under global $PSU(2,2|4)$ transformations which act by left multiplication on the coset supergroup elements and it is invariant under local $SO(4,1) \times SO(5)$ gauge transformations which act by right multiplication on g.[1] The coupling constant is $\alpha = (\lambda)^{-\frac{1}{4}} = (Ng_s)^{-\frac{1}{4}}$ and the coefficients of the action are fixed in such a way that the action is gauge invariant under local $SO(4,1) \times SO(5)$ transformations, according to the metric and the structure constant normalized as in the appendix.

Using the background field method [9, 3, 10], one can compute the one-loop effective action. The mapping g is parameterized as a classical background plus quantum fluctuations around the background: $g = \tilde{g}e^{\alpha X}$. The gauge invariance of the original action can be used to fix $X \in \mathcal{G}/\mathcal{H}_0$. Plugging $g = \tilde{g}e^{\alpha X}$ in J, $\overline{J}$ and expanding up to the α^2 order,

[1] Under a local gauge $SO(4,1) \times SO(5)$ transformation with parameter $\xi \in \mathcal{H}_0$, J_i and $\overline{J}_i$ transform as $\delta J_i = [J_i, \xi]$, $\delta\overline{J}_i = [\overline{J}_i, \xi]$, while J_0 and $\overline{J}_0$ transform as a connection $\delta J_0 = \partial\xi + [J_0, \xi]$, $\delta\overline{J}_0 = \overline{\partial}\xi + [\overline{J}_0, \xi]$, the ghost currents as $\delta N = [N, \xi]$, $\delta\hat{N} = [\hat{N}, \xi]$, and the pure spinors and their conjugate momenta transform as $\delta\lambda = [\lambda, \xi]$, $\delta\omega = [\omega, \xi]$, analogously for the hatted spinors, [5, 12].

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the matter currents are given in terms of the X fields by:

$$
\begin{aligned}
J_i &= \widetilde{J}_i + \alpha(\partial X_i + [\widetilde{J}, X]_i) + \frac{\alpha^2}{2}([[\widetilde{J}, X], X]_i + [\partial X, X]_i) + \dots \\
\overline{J}_i &= \widetilde{\overline{J}}_i + \alpha(\overline{\partial} X_i + [\widetilde{\overline{J}}, X]_i) + \frac{\alpha^2}{2}([[\widetilde{\overline{J}}, X], X]_i + [\overline{\partial} X, X]_i) + \dots,
\end{aligned}
\tag{2.2}
$$

where $i = 1, 2, 3$ labels the elements of the subalgebras $\mathcal{H}_i$, i.e. $J_i \equiv J_{|\mathcal{H}_i}$, $\widetilde{J}$ and $\widetilde{\overline{J}}$ are the classical currents, $\widetilde{J} = \tilde{g}^{-1}\partial\tilde{g}$, $\widetilde{\overline{J}} = \tilde{g}\overline{\partial}\tilde{g}$.
Though $X \in \mathcal{G}/\mathcal{H}_0$, the fluctuations can contain the gauge fields since the commutators in (2.2) can contain J_0 and $\overline{J}_0$. Thanks to the gauge invariance, the effective action is independent of them [9], thus they can be gauged away, i.e. $[J_0, X_i] = [\overline{J}_0, X_i] = 0$ for any X_i. Furthermore J_0 and $\overline{J}_0$ can have quantum fluctuations according to:

$$
\begin{aligned}
J_0 &= \tilde{J}_0 + \alpha[\tilde{J}, X]_0 + \frac{\alpha^2}{2}([\partial X, X]_0 + [[\tilde{J}, X], X]_0) + \dots, \\
\overline{J}_0 &= \tilde{\overline{J}}_0 + \alpha[\tilde{\overline{J}}, X]_0 + \frac{\alpha^2}{2}([\overline{\partial} X, X]_0 + [[\tilde{\overline{J}}, X], X]_0) + \dots.
\end{aligned}
\tag{2.3}
$$

We will treat the Lorentz ghost currents as external ones.

Since we want to know the tree-level OPE for the matter currents, we need to compute the action for the X fluctuations only to the first order in the external currents. Plugging the expansion of the currents (2.2) and (2.3) in the action (2.1), one gets terms of zeroth order in the X fields, which are the action for the background fields, linear terms in X, which vanish by general arguments of QFT, and second order terms in the fluctuations. Thus keeping all the terms of α^2 order and neglecting all the contributions which are of the second order in the classical currents,[2] one obtains the following action:

$$
\begin{aligned}
S = &\int d^2z\{-\eta_{\underline{ab}} X^{\underline{a}}\partial\overline{\partial}X^{\underline{b}} - \eta_{\alpha\hat{\beta}}X^{\alpha}\partial\overline{\partial}X^{\hat{\beta}} - \eta_{\hat{\beta}\alpha}X^{\hat{\beta}}\partial\overline{\partial}X^{\alpha}\}+ \\
+ &\int d^2z\Big\{X^{\alpha}\Big[\frac{1}{2}\eta_{\underline{ab}}f^{\underline{b}}_{\alpha\beta}(\overleftarrow{\overline{\partial}}\, J^{\underline{a}} + J^{\underline{a}}\overrightarrow{\overline{\partial}})\Big]X^{\beta} + X^{\hat{\alpha}}\Big[\frac{1}{2}\eta_{\underline{ab}}f^{\underline{a}}_{\hat{\alpha}\hat{\beta}}(\overleftarrow{\partial}\,\overline{J}^{\underline{b}} + \overline{J}^{\underline{b}}\overrightarrow{\partial})\Big]X^{\hat{\beta}}+ \\
&+ X^{\underline{a}}[\eta_{\alpha\hat{\beta}}f^{\hat{\beta}}_{\underline{a}\beta}(\overleftarrow{\overline{\partial}}\, J^{\alpha} + J^{\alpha}\overrightarrow{\overline{\partial}})]X^{\beta} + X^{\underline{a}}[\eta_{\hat{\alpha}\beta}f^{\beta}_{\underline{a}\hat{\beta}}(\overleftarrow{\partial}\,\overline{J}^{\hat{\alpha}} + \overline{J}^{\hat{\alpha}}\overrightarrow{\partial})]X^{\hat{\beta}}+ \\
&+ X^{\underline{a}}\Big[-\frac{1}{4}f^{[\underline{cd}]}_{\underline{ab}}(\overleftarrow{\overline{\partial}}\, N_{\underline{cd}} + N_{\underline{cd}}\overrightarrow{\overline{\partial}}) - \frac{1}{4}f^{[\underline{cd}]}_{\underline{ab}}(\overleftarrow{\partial}\,\hat{N}_{\underline{cd}} + \hat{N}_{\underline{cd}}\overrightarrow{\partial})\Big]X^{\underline{b}}+ \\
&+ X^{\alpha}\Big[-\frac{1}{2}f^{\underline{cd}}_{\alpha\hat{\beta}}(\overleftarrow{\overline{\partial}}\, N_{\underline{cd}} + N_{\underline{cd}}\overrightarrow{\overline{\partial}}) - \frac{1}{2}f^{\underline{cd}}_{\alpha\hat{\beta}}(\overline{\partial}\hat{N}_{\underline{cd}} + \hat{N}_{\underline{cd}}\overline{\partial})\Big]X^{\hat{\beta}}\Big\}
\end{aligned}
\tag{2.4}
$$

All the currents that are present in the action (2.4) are the classical ones (the ˜ is omitted in the notation on what follows). From a diagrammatic point of view this means considering all the tree-level diagrams, i.e. with one insertion of the external current, either matter current J^A, $\overline{J}^A$ or Lorentz ghost current $N_{\underline{cd}}$, $\hat{N}_{\underline{cd}}$.

[2]For a dimensional analysis this implies neglecting also the terms of order ∂J.

3. The tree-level interactions

In order to read off the propagators for the quantum fluctuations one has to invert perturbatively the operator between the X's. The kinetic term is given by the (super) matrix:

$$A = \begin{bmatrix} \eta_{\underline{ab}}(-\partial\overline{\partial}) & 0 & 0 \\ 0 & 0 & \eta_{\alpha\hat{\beta}}(-\partial\overline{\partial}) \\ 0 & \eta_{\hat{\alpha}\beta}(-\partial\overline{\partial}) & 0 \end{bmatrix}. \tag{3.1}$$

The inverse of this matrix is given by:

$$A^{-1} = \begin{bmatrix} \eta^{\underline{ab}}(-\partial\overline{\partial})^{-1} & 0 & 0 \\ 0 & 0 & \eta^{\beta\hat{\alpha}}(-\partial\overline{\partial})^{-1} \\ 0 & \eta^{\hat{\beta}\alpha}(-\partial\overline{\partial})^{-1} & 0 \end{bmatrix}. \tag{3.2}$$

where $(-\partial\overline{\partial})^{-1}$ is formally the "free" propagators, namely $(-\partial\overline{\partial})^{-1} = -\frac{1}{2\pi}\log|y-z|^2$. The coefficient in front of the propagator is fixed by the differential equation $\partial\overline{\partial}\log|z|^2 = 2\pi\delta^{(2)}(z,\overline{z})$.[3] In this way the integral of the δ function is normalized to 1, $\int d^2z\delta^{(2)}(z,\overline{z}) = 1$.

If the operator can be represented as a sum of two matrices A and V, the inverse of the matrix is perturbatively:

$$(A+V)^{-1} = A^{-1} - (A^{-1}VA^{-1}) + \dots \tag{3.3}$$

where V is the matrix containing the tree-level interaction with the matter and the Lorentz ghost currents. The entries of the matrix V are just the terms containing the currents in the action (2.4), the terms off-diagonal divided by 1/2:

$$\begin{aligned}
V_{11} &= -\frac{1}{4}(f_{\underline{ab}}^{[\underline{cd}]}(\overleftarrow{\overline{\partial}}\, N_{\underline{cd}} + N_{\underline{cd}}\,\overrightarrow{\overline{\partial}}) + f_{\underline{ab}}^{[\underline{cd}]}(\overleftarrow{\partial}\,\hat{N}_{\underline{cd}} + \hat{N}_{\underline{cd}}\,\overrightarrow{\partial})) \\
V_{12} &= \frac{1}{2}\eta_{\rho\hat{\rho}}f_{\underline{a}\beta}^{\hat{\rho}}(\overleftarrow{\overline{\partial}}\, J^{\rho} + J^{\rho}\,\overrightarrow{\overline{\partial}}) \\
V_{13} &= -\frac{1}{2}\eta_{\rho\hat{\rho}}f_{\underline{a}\hat{\beta}}^{\rho}(\overleftarrow{\partial}\,\overline{J}^{\hat{\rho}} + \overline{J}^{\hat{\rho}}\,\overrightarrow{\partial}) \\
V_{22} &= \frac{1}{2}\eta_{\underline{ab}}f_{\alpha\beta}^{\underline{b}}(\overleftarrow{\overline{\partial}}\, J^{\underline{a}} + J^{\underline{a}}\,\overrightarrow{\overline{\partial}}) \\
V_{23} &= -\frac{1}{4}(f_{\alpha\hat{\beta}}^{\underline{cd}}(\overleftarrow{\overline{\partial}}\, N_{\underline{cd}} + N_{\underline{cd}}\,\overrightarrow{\overline{\partial}}) + f_{\alpha\hat{\beta}}^{\underline{cd}}(\overleftarrow{\partial}\,\hat{N}_{\underline{cd}} + \hat{N}_{\underline{cd}}\,\overrightarrow{\partial})) \\
V_{33} &= \frac{1}{2}\eta_{\underline{ab}}f_{\hat{\alpha}\hat{\beta}}^{\underline{a}}(\overleftarrow{\partial}\,\overline{J}^{\underline{b}} + \overline{J}^{\underline{b}}\,\overrightarrow{\partial}).
\end{aligned} \tag{3.4}$$

4. OPE

The general expression for the OPE of the currents is at the order considered:

$$\begin{aligned}
J^A(y)J^B(z) = &< \widetilde{J}^A(y)\widetilde{J}^B(z) > +\alpha^2\Big(< \partial X^A(y)\partial X^B(z) > + \\
&+ < \partial X^A(y)[\widetilde{J},X]^B(z) > + < [\widetilde{J},X]^A(y)\partial X^B(z) > +\mathcal{O}(J^2)\Big).
\end{aligned} \tag{4.1}$$

[3] The δ function in the complex plane is normalized as in [13].

The currents are taken normal-ordered to avoid the contractions on the same points. The classical terms given by the propagator of two currents in (4.1) will be not considered and the contractions on the last two terms in (4.1) must be done keeping in mind that these contributions are already at the tree-level order, namely they already contain an external leg. On what follows all the currents that appear in the r.h.s. of the OPE are the classical ones. The results are proportional to the coupling constant α^2, this overall factor is omitted in the final result as well as the classical term.

$$\begin{aligned} J^{\underline{a}}(y)J^{\underline{d}}(0) &\simeq (cl.) + \alpha^2(<\partial X^{\underline{a}}(y)\partial X^{\underline{d}}(0)> + \ldots) \simeq \\ &\simeq -\frac{\eta^{\underline{ad}}}{2\pi}\frac{1}{y^2} + \frac{1}{4\pi}\eta^{[\underline{ad}][\underline{ef}]}\hat{N}_{\underline{ef}}(0)\frac{\overline{y}}{y^2} - \frac{1}{4\pi y}\eta^{[\underline{ad}][\underline{ef}]}N_{\underline{ef}}(0). \end{aligned} \tag{4.2}$$

$$\begin{aligned} J^{\underline{a}}(y)\overline{J}^{\underline{d}}(0) &\simeq (cl.) + \alpha^2(<\partial X^{\underline{a}}(y)\overline{\partial} X^{\underline{d}}(0)> + \ldots) \simeq \\ &\simeq -\frac{1}{4\pi\overline{y}}\eta^{[\underline{ad}][\underline{ef}]}N_{\underline{ef}}(0) - \frac{1}{4\pi y}\eta^{[\underline{ad}][\underline{ef}]}\hat{N}_{\underline{ef}}(0). \end{aligned} \tag{4.3}$$

$$\begin{aligned} \overline{J}^{\underline{a}}(y)J^{\underline{d}}(0) &\simeq (cl.) + \alpha^2(<\overline{\partial} X^{\underline{a}}(y)\partial X^{\underline{d}}(0)> + \ldots) \simeq \\ &\simeq -\frac{1}{4\pi\overline{y}}\eta^{[\underline{ad}][\underline{ef}]}N_{\underline{ef}}(0) - \frac{1}{4\pi y}\eta^{[\underline{ad}][\underline{ef}]}\hat{N}_{\underline{ef}}(0). \end{aligned} \tag{4.4}$$

$$\begin{aligned} \overline{J}^{\underline{a}}(y)\overline{J}^{\underline{d}}(0) &\simeq (cl.) + \alpha^2(<\overline{\partial} X^{\underline{a}}(y)\partial X^{\underline{d}}(0)> + \ldots) \simeq \\ &\simeq -\frac{1}{2\pi\overline{y}^2}\eta^{\underline{ad}} + \frac{1}{4\pi}\frac{y}{\overline{y}^2}\eta^{[\underline{ad}][\underline{ef}]}N_{\underline{ef}}(0) - \frac{1}{4\pi\overline{y}}\eta^{[\underline{ad}][\underline{ef}]}\hat{N}_{\underline{ef}}(0). \end{aligned} \tag{4.5}$$

$$\begin{aligned} J^{\underline{a}}(y)J^{\delta}(0) &\simeq \alpha^2(<\partial X^{\underline{a}}(y)\partial X^{\delta}(0)> + <\partial X^{\underline{a}}(y)[J_3, X_2]^{\delta}(0)> + \\ &+ <[J_3, X_3]^{\underline{a}}(y)\partial X^{\delta}(0)> + \ldots) \simeq \frac{1}{2\pi}\frac{\overline{y}}{y^2}f^{\underline{a}}_{\hat{\gamma}\hat{\rho}}\eta^{\hat{\gamma}\delta}\overline{J}^{\hat{\rho}}(0) + \frac{1}{\pi y}f^{\underline{a}}_{\hat{\rho}\hat{\gamma}}\eta^{\hat{\gamma}\delta}J^{\hat{\rho}}(0). \end{aligned} \tag{4.6}$$

$$\begin{aligned} J^{\underline{a}}(y)\overline{J}^{\delta}(0) &\simeq \alpha^2(<\partial X^{\underline{a}}(y)\overline{\partial} X^{\delta}(0)> + <\partial X^{\underline{a}}(y)[\overline{J}_3, X_2]^{\delta}(0)> + \\ &+ <[J_3, X_3]^{\underline{a}}(y)\overline{\partial} X^{\delta}(0)> + \ldots) \simeq \frac{1}{2\pi\overline{y}}f^{\underline{a}}_{\hat{\gamma}\hat{\rho}}\eta^{\hat{\gamma}\delta}J^{\hat{\rho}}(0). \end{aligned} \tag{4.7}$$

$$\begin{aligned} \overline{J}^{\underline{a}}(y)J^{\delta}(0) &\simeq \alpha^2(<\overline{\partial} X^{\underline{a}}(y)\partial X^{\delta}(0)> + <\overline{\partial} X^{\underline{a}}(y)[J_3, X_2]^{\delta}(0)> + \\ &+ <[\overline{J}_3, X_3]^{\underline{a}}(y)\partial X^{\delta}(0)> + \ldots) \simeq \frac{1}{2\pi\overline{y}}f^{\underline{a}}_{\hat{\gamma}\hat{\rho}}\eta^{\hat{\gamma}\delta}J^{\hat{\rho}}(0). \end{aligned} \tag{4.8}$$

$$\begin{aligned} \overline{J}^{\underline{a}}(y)\overline{J}^{\delta}(0) &\simeq \alpha^2(<\overline{\partial} X^{\underline{a}}(y)\overline{\partial} X^{\delta}(0)> + <\overline{\partial} X^{\underline{a}}(y)[\overline{J}_3, X_2]^{\delta}(0)> + \\ &+ <[\overline{J}_3, X_3]^{\underline{a}}(y)\overline{\partial} X^{\delta}(0)> + \ldots) \simeq \frac{1}{2\pi\overline{y}}f^{\underline{a}}_{\hat{\gamma}\hat{\rho}}\eta^{\hat{\gamma}\delta}\overline{J}^{\hat{\rho}}(0). \end{aligned} \tag{4.9}$$

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$$
\begin{aligned}
J^{\underline{a}}(y)J^{\hat{\delta}}(0) \simeq{}& \alpha^2(<\partial X^{\underline{a}}(y)\partial X^{\hat{\delta}}(0)> + <\partial X^{\underline{a}}(y)[J_1,X_2]^{\hat{\delta}}(0)> + \\
&+ <[J_1,X_1]^{\underline{a}}(y)\partial X^{\hat{\delta}}(0)> + \ldots) \simeq \frac{1}{2\pi y} f^{\underline{a}}_{\rho\gamma}\eta^{\gamma\hat{\delta}}J^{\rho}(0).
\end{aligned} \tag{4.10}
$$

$$
\begin{aligned}
J^{\underline{a}}(y)\overline{J}^{\hat{\delta}}(0) \simeq{}& \alpha^2(<\partial X^{\underline{a}}(y)\overline{\partial} X^{\hat{\delta}}(0)> + <\partial X^{\underline{a}}(y)[\overline{J}_1,X_2]^{\hat{\delta}}(0)> + \\
&+ <[J_1,X_1]^{\underline{a}}(y)\overline{\partial} X^{\hat{\delta}}(0)> + \ldots) \simeq \frac{1}{2\pi y} f^{\underline{a}}_{\rho\gamma}\eta^{\gamma\hat{\delta}}\overline{J}^{\rho}(0).
\end{aligned} \tag{4.11}
$$

$$
\begin{aligned}
\overline{J}^{\underline{a}}(y)J^{\hat{\delta}}(0) \simeq{}& \alpha^2(<\overline{\partial} X^{\underline{a}}(y)\partial X^{\hat{\delta}}(0)> + <\overline{\partial} X^{\underline{a}}(y)[J_1,X_2]^{\hat{\delta}}(0)> + \\
&+ <[\overline{J}_1,X_1]^{\underline{a}}(y)\partial X^{\hat{\delta}}(0)> + \ldots) \simeq \frac{1}{2\pi y} f^{\underline{a}}_{\rho\gamma}\eta^{\gamma\hat{\delta}}\overline{J}^{\rho}(0).
\end{aligned} \tag{4.12}
$$

$$
\begin{aligned}
\overline{J}^{\underline{a}}(y)\overline{J}^{\hat{\delta}}(0) \simeq{}& \alpha^2(<\overline{\partial} X^{\underline{a}}(y)\overline{\partial} X^{\hat{\delta}}(0)> + <\overline{\partial} X^{\underline{a}}(y)[\overline{J}_1,X_2]^{\hat{\delta}}(0)> + \\
&+ <[\overline{J}_1,X_1]^{\underline{a}}(y)\overline{\partial} X^{\hat{\delta}}(0)> + \ldots) \simeq \frac{1}{2\pi}\frac{y}{\overline{y}^2} f^{\underline{a}}_{\rho\gamma}\eta^{\gamma\hat{\delta}}J^{\rho}(0) + \frac{1}{\pi\overline{y}} f^{\underline{a}}_{\rho\gamma}\eta^{\gamma\hat{\delta}}\overline{J}^{\rho}(0).
\end{aligned} \tag{4.13}
$$

$$
\begin{aligned}
J^{\alpha}(y)J^{\delta}(0) \simeq{}& \alpha^2(<\partial X^{\alpha}(y)\partial X^{\delta}(0)> + <\partial X^{\alpha}(y)[J_2,X_3]^{\delta}(0)> + \\
&+ <[J_2,X_3]^{\alpha}(y)\partial X^{\delta}(0)> + \ldots) \simeq \frac{1}{2\pi}\frac{\overline{y}}{y^2} f^{\alpha}_{\underline{l}\hat{\gamma}}\eta^{\hat{\gamma}\delta}\overline{J}^{\underline{l}}(0) + \frac{1}{\pi y} f^{\alpha}_{\underline{l}\hat{\gamma}}\eta^{\hat{\gamma}\delta}J^{\underline{l}}(0).
\end{aligned} \tag{4.14}
$$

$$
\begin{aligned}
\overline{J}^{\alpha}(y)J^{\delta}(0) \simeq{}& \alpha^2(<\overline{\partial} X^{\alpha}(y)\partial X^{\delta}(0)> + <\overline{\partial} X^{\alpha}(y)[J_2,X_3]^{\delta}(0)> + \\
&+ <[\overline{J}_2,X_3]^{\alpha}(y)\partial X^{\delta}(0)> + \ldots) \simeq \frac{1}{2\pi\overline{y}} f^{\alpha}_{\underline{l}\hat{\gamma}}\eta^{\hat{\gamma}\delta}J^{\underline{l}}(0).
\end{aligned} \tag{4.15}
$$

$$
\begin{aligned}
J^{\alpha}(y)\overline{J}^{\delta}(0) \simeq{}& \alpha^2(<\partial X^{\alpha}(y)\overline{\partial} X^{\delta}(0)> + <\partial X^{\alpha}(y)[\overline{J}_2,X_3]^{\delta}(0)> + \\
&+ <[J_2,X_3]^{\alpha}(y)\overline{\partial} X^{\delta}(0)> + \ldots) \simeq \frac{1}{2\pi\overline{y}} f^{\alpha}_{\underline{l}\hat{\gamma}}\eta^{\hat{\gamma}\delta}J^{\underline{l}}(0).
\end{aligned} \tag{4.16}
$$

$$
\begin{aligned}
\overline{J}^{\alpha}(y)\overline{J}^{\delta}(0) \simeq{}& \alpha^2(<\overline{\partial} X^{\alpha}(y)\overline{\partial} X^{\delta}(z)> + <\overline{\partial} X^{\alpha}(y)[\overline{J}_2,X_3]^{\delta}(0)> + \\
&+ <[\overline{J}_2,X_3]^{\alpha}(y)\overline{\partial} X^{\delta}(0)> + \ldots) \simeq \frac{1}{2\pi\overline{y}} f^{\alpha}_{\underline{l}\hat{\gamma}}\eta^{\hat{\gamma}\delta}\overline{J}^{\underline{l}}(0).
\end{aligned} \tag{4.17}
$$

$$
\begin{aligned}
J^{\hat{\alpha}}(y)J^{\hat{\delta}}(0) \simeq{}& \alpha^2(<\partial X^{\hat{\alpha}}(y)\partial X^{\hat{\delta}}(0)> + <\partial X^{\hat{\alpha}}(y)[J_2,X_1]^{\hat{\delta}}(0)> + \\
&+ <[J_2,X_1]^{\hat{\alpha}}(y)\partial X^{\hat{\delta}}(0)> + \ldots) \simeq \frac{1}{2\pi y} f^{\hat{\alpha}}_{\underline{l}\gamma}\eta^{\gamma\hat{\delta}}J^{\underline{l}}(0).
\end{aligned} \tag{4.18}
$$

$$
\begin{aligned}
\overline{J}^{\hat{\alpha}}(y)J^{\hat{\delta}}(0) \simeq{}& \alpha^2(<\overline{\partial} X^{\hat{\alpha}}(y)\partial X^{\hat{\delta}}(0)> + <\overline{\partial} X^{\hat{\alpha}}(y)[J_2,X_1]^{\hat{\delta}}(0)> + \\
&+ <[\overline{J}_2,X_1]^{\hat{\alpha}}(y)\partial X^{\hat{\delta}}(0)> + \ldots) \simeq \frac{1}{2\pi y} f^{\hat{\alpha}}_{\underline{l}\gamma}\eta^{\gamma\hat{\delta}}\overline{J}^{\underline{l}}(0).
\end{aligned} \tag{4.19}
$$

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$$J^{\hat{\alpha}}(y)\overline{J}^{\hat{\hat{\delta}}}(0) \simeq \alpha^2(<\partial X^{\hat{\alpha}}(y)\overline{\partial} X^{\hat{\hat{\delta}}}(0)> + <\partial X^{\hat{\alpha}}(y)[\overline{J}_2, X_1]^{\hat{\hat{\delta}}}(0)> + \\ + <[J_2, X_1]^{\hat{\alpha}}(y)\overline{\partial} X^{\hat{\hat{\delta}}}(0)> + \ldots) \simeq \frac{1}{2\pi y} f^{\hat{\alpha}}_{\underline{l}\gamma}\eta^{\gamma\hat{\hat{\delta}}}\overline{J}^{\underline{l}}(0). \tag{4.20}$$

$$\overline{J}^{\hat{\alpha}}(y)\overline{J}^{\hat{\hat{\delta}}}(0) \simeq \alpha^2(<\overline{\partial} X^{\hat{\alpha}}(y)\overline{\partial} X^{\hat{\hat{\delta}}}(0)> + <\overline{\partial} X^{\hat{\alpha}}(y)[\overline{J}_2, X_1]^{\hat{\hat{\delta}}}(0)> + \\ + <[\overline{J}_2, X_1]^{\hat{\alpha}}(y)\overline{\partial} X^{\hat{\hat{\delta}}}(0)> + \ldots) \simeq \frac{1}{2\pi}\frac{y}{\overline{y}^2} f^{\hat{\alpha}}_{\underline{l}\gamma}\eta^{\gamma\hat{\hat{\delta}}} J^{\underline{l}}(0) + \frac{1}{\pi\overline{y}} f^{\hat{\alpha}}_{\underline{l}\gamma}\eta^{\gamma\hat{\hat{\delta}}}\overline{J}^{\underline{l}}(0). \tag{4.21}$$

$$J^{\hat{\alpha}}(y)J^{\delta}(0) \simeq (cl.) + \alpha^2 <\partial X^{\hat{\alpha}}(y)\partial X^{\delta}(0)> + \ldots \simeq \\ \simeq -\frac{1}{2\pi y^2}\eta^{\hat{\alpha}\delta} + \frac{1}{4\pi}\frac{\overline{y}}{y^2}\eta^{\hat{\alpha}\beta} f^{[ef]}_{\beta\hat{\gamma}}\eta^{\hat{\gamma}\delta}\hat{N}_{\underline{ef}}(0) - \frac{1}{4\pi y}\eta^{\hat{\alpha}\beta} f^{[ef]}_{\beta\hat{\gamma}}\eta^{\hat{\gamma}\delta} N_{\underline{ef}}(0). \tag{4.22}$$

$$\overline{J}^{\hat{\alpha}}(y)J^{\delta}(0) \simeq (cl.) + \alpha^2 <\overline{\partial} X^{\hat{\alpha}}(y)\partial X^{\delta}(0)> + \ldots \simeq \\ \simeq -\frac{1}{4\pi\overline{y}}\eta^{\hat{\alpha}\beta} f^{[ef]}_{\beta\hat{\gamma}}\eta^{\hat{\gamma}\delta} N_{\underline{ef}}(0) - \frac{1}{4\pi y}\eta^{\hat{\alpha}\beta} f^{[ef]}_{\beta\hat{\gamma}}\eta^{\hat{\gamma}\delta}\hat{N}_{\underline{ef}}(0). \tag{4.23}$$

$$J^{\hat{\alpha}}(y)\overline{J}^{\delta}(0) \simeq (cl.) + \alpha^2 <\partial X^{\hat{\alpha}}(y)\overline{\partial} X^{\delta}(0)> + \ldots \simeq \\ \simeq -\frac{1}{4\pi\overline{y}}\eta^{\hat{\alpha}\beta} f^{[ef]}_{\beta\hat{\gamma}}\eta^{\hat{\gamma}\delta} N_{\underline{ef}}(0) - \frac{1}{4\pi y}\eta^{\hat{\alpha}\beta} f^{[ef]}_{\beta\hat{\gamma}}\eta^{\hat{\gamma}\delta}\hat{N}_{\underline{ef}}(0). \tag{4.24}$$

$$\overline{J}^{\hat{\alpha}}(y)\overline{J}^{\delta}(0) \simeq (cl.) + \alpha^2 <\overline{\partial} X^{\hat{\alpha}}(y)\overline{\partial} X^{\delta}(0)> + \ldots \simeq \\ \simeq -\frac{1}{2\pi\overline{y}^2}\eta^{\hat{\alpha}\delta} + \frac{1}{4\pi}\frac{y}{\overline{y}^2}\eta^{\hat{\alpha}\beta} f^{[ef]}_{\beta\hat{\gamma}}\eta^{\hat{\gamma}\delta} N_{\underline{ef}}(0) - \frac{1}{4\pi\overline{y}}\eta^{\hat{\alpha}\beta} f^{[ef]}_{\beta\hat{\gamma}}\eta^{\hat{\gamma}\delta}\hat{N}_{\underline{ef}}(0). \tag{4.25}$$

All the current OPEs respect the Z_4-grading of the $psu(2,2|4)$ super-algebra. In fact the OPE of two currents with indices A and B is proportional to a current with index $C = A+B$ (mod 4). Of course this reflects the fact that the tree-level interactions between the X fields respect the Z_4-automorphism of the super-algebra, since the couplings which we can obtain are allowed by the matrices (3.1) and (3.4).

Using the above results we have checked that the OPEs between the currents and the classical equations of motion[4] derived from (2.1) vanish.
We have checked also that the OPEs found here reproduce commutators of the currents computed in [14] after the Wick rotation to the Minkowskian world-sheet. Because of our gauge choice J_0 and $\overline{J}_0$ are absent in the commutators and the constraints, which are present in [14], vanish.

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[4]The equations of motion are listed in the appendix C.

A. Notation

The $psu(2,2|4)$ super Lie algebra has a special inner symmetry, the so-called $\mathbf{Z}_4$-automorphism [9], that allows to decompose it in

$$\mathcal{G} = \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \mathcal{H}_3. \tag{A.1}$$

The space $\mathcal{H}_k$ is the eigenspace with respect to the Z_4 action and the corresponding eigenvalue is $\imath^k$. Thus $\mathcal{H}_0$ is the locus of fixed points with respect the Z_4 transformation. Since the Z_4-grading is an automorphism of the super Lie algebra, the decomposition (A.1) respects the structure of the algebra, i.e. satisfies $[\mathcal{H}_m, \mathcal{H}_n] \subset \mathcal{H}_{m+n \pmod 4}$ and also the bilinear form is Z_4-invariant:

$$< \mathcal{H}_m, \mathcal{H}_n >= 0 \qquad unless \;\; m+n = 0 \pmod 4 \tag{A.2}$$

The subalgebra $\mathcal{H}_0$ is exactly the invariant subalgebra for the gauge $SO(4,1) \times SO(5)$ group. The $\mathcal{H}_2$ subalgebra is the space for the remaining bosonic elements (it contains the "translation" generators), while $\mathcal{H}_1$ and $\mathcal{H}_3$ contain the fermionic elements ("supersymmetry" generators).
Therefore the generators of the super-algebra are decomposed in:

$$T_{\underline{a}} \in \mathcal{H}_2 \qquad T_\alpha \in \mathcal{H}_1 \qquad T_{\hat{\alpha}} \in \mathcal{H}_3 \qquad T_{[\underline{cd}]} \in \mathcal{H}_0, \tag{A.3}$$

and consequently the currents:

$$\begin{aligned} J &= g^{-1}\partial g = J^A T_A = J^{\underline{a}} T_{\underline{a}} + J^\alpha T_\alpha + J^{\hat{\alpha}} T_{\hat{\alpha}} + J^{[\underline{cd}]} T_{[\underline{cd}]} \\ \overline{J} &= g^{-1}\overline{\partial} g = \overline{J}^A T_A = \overline{J}^{\underline{a}} T_{\underline{a}} + \overline{J}^\alpha T_\alpha + \overline{J}^{\hat{\alpha}} T_{\hat{\alpha}} + \overline{J}^{[\underline{cd}]} T_{[\underline{cd}]}. \end{aligned} \tag{A.4}$$

The indices $A = (\underline{a}, [\underline{cd}], \alpha, \hat{\alpha})$ label the tangent spaces of the super Lie algebra; in particular $\underline{a} = (a, a')$, $a = 0, \ldots, 4$ labels the $so(4,1)$ vector index for AdS_5, $a' = 5, \ldots, 9$ labels the $so(5)$ vector index for S^5, $[\underline{cd}] = ([cd], [c'd'])$ and α, $\hat{\alpha} = 1, ..., 16$ label the two sixteen-component Majorana-Weyl spinors in $D = 10$.
In the curved background the two fermionic indices can couple thanks to the matrix $\delta_{\alpha\hat{\alpha}} = (\gamma^{01234})_{\alpha\hat{\alpha}}$, with $0,1,2,3,4$ the directions of AdS_5.

The super-trace is cyclic up to a minus sign, i.e.

$$Str(XY) = (-1)^{deg(X)deg(Y)} Str(YX), \tag{A.5}$$

where $deg(X) = 0$ if X is even and $deg(X) = 1$ if X is odd. This is consistent with the statistic. The relation[5]

$$Str(T_A[T_B, T_C]) = Str([T_A, T_B]T_C), \tag{A.6}$$

[5]The commutator has to be understood as a graded commutator: $[T_A, T_B] = T_A T_B - (-1)^{|A||B|} T_B T_A$, where $|A| = 1$ for odd generators and $|A| = 0$ for even generators.

furnishes some important graded properties for the structure constants. The non-vanishing structure constants for the $psu(2,2|4)$ super-algebra are the following:

$$\begin{gathered}
f_{\alpha\hat{\beta}}^{[ab]} = \frac{1}{2}(\gamma^{ab})_{\alpha}{}^{\gamma}\delta_{\gamma\hat{\beta}} \qquad f_{\alpha\hat{\beta}}^{[a'b']} = -\frac{1}{2}(\gamma^{a'b'})_{\alpha}{}^{\gamma}\delta_{\gamma\hat{\beta}} \qquad f_{[\underline{cd}]\beta}^{\alpha} = -f_{\beta[\underline{cd}]}^{\alpha} = \frac{1}{2}(\gamma_{\underline{cd}})_{\beta}{}^{\alpha} \\
f_{[\underline{cd}]\hat{\beta}}^{\hat{\alpha}} = -f_{\hat{\beta}[\underline{cd}]}^{\hat{\alpha}} = \frac{1}{2}(\gamma_{\underline{cd}})_{\hat{\beta}}{}^{\hat{\alpha}} \qquad f_{\alpha\beta}^{\underline{a}} = f_{\beta\alpha}^{\underline{a}} = \gamma_{\alpha\beta}^{\underline{a}} \qquad f_{\underline{a}\beta}^{\hat{\beta}} = -f_{\beta\underline{a}}^{\hat{\beta}} = -(\gamma_{\underline{a}})_{\beta\gamma}\delta^{\gamma\hat{\beta}} \\
f_{\hat{\alpha}\hat{\beta}}^{\underline{a}} = f_{\hat{\alpha}\hat{\beta}}^{\underline{a}} = \gamma_{\hat{\alpha}\hat{\beta}}^{\underline{a}} \qquad f_{\underline{a}\hat{\alpha}}^{\alpha} = -f_{\hat{\alpha}\underline{a}}^{\alpha} = (\gamma_{\underline{a}})_{\hat{\alpha}\hat{\beta}}\delta^{\alpha\hat{\beta}} \qquad f_{ab}^{[ef]} = -f_{ba}^{[ef]} = \delta_{a}^{[e}\delta_{b}^{f]} \\
f_{a'b'}^{[e'f']} = -f_{b'a'}^{[e'f']} = -\delta_{a'}^{[e'}\delta_{b'}^{f']} \qquad f_{[\underline{cd}]\underline{b}}^{\underline{e}} = -f_{\underline{b}[\underline{cd}]}^{\underline{e}} = \eta_{\underline{b}[\underline{c}}\delta_{\underline{d}]}^{\underline{e}} \\
f_{[\underline{cd}][\underline{ef}]}^{[\underline{gh}]} = \eta_{\underline{ce}}\delta_{\underline{d}}^{[\underline{g}}\delta_{\underline{f}}^{\underline{h}]} - \eta_{\underline{cf}}\delta_{\underline{d}}^{[\underline{g}}\delta_{\underline{e}}^{\underline{h}]} + \eta_{\underline{df}}\delta_{\underline{c}}^{[\underline{g}}\delta_{\underline{e}}^{\underline{h}]} - \eta_{\underline{de}}\delta_{\underline{c}}^{[\underline{g}}\delta_{\underline{f}}^{\underline{h}]}
\end{gathered} \tag{A.7}$$

The metric η_{AB} is given by:

$$\eta_{\underline{ab}} \qquad \eta_{\alpha\hat{\beta}} = -\eta_{\hat{\beta}\alpha} = \delta_{\alpha\hat{\beta}} \qquad \eta_{[a'b'][c'd']} = -\eta_{a'[c'}\eta_{d']b'} \qquad \eta_{[ab][cd]} = \eta_{a[c}\eta_{d]b} \tag{A.8}$$

Furthermore if a super-matrix is defined as:

$$K = \begin{bmatrix} A & C \\ D & B \end{bmatrix}, \tag{A.9}$$

with A and B even matrices and C and D odd, then the super-transpose is given by:

$$K^{ST} = \begin{bmatrix} A^T & -D^T \\ C^T & B^T \end{bmatrix}. \tag{A.10}$$

A.1 Gamma matrices in $D = 10$ dimensions

In $D = 10$ dimensions in the reducible Majorana-Weyl representations the (32×32) Dirac gamma matrices $\Gamma_{AB}^{\underline{m}}$ are real and symmetric and they consist of two symmetric 16×16 matrices $\gamma_{\alpha\beta}^{\underline{m}}$, $\gamma^{\underline{m}\,\alpha\beta}$ on the off-diagonal.[6]

$$\Gamma^{\underline{m}} = \begin{bmatrix} 0 & \gamma^{\underline{m}\,\alpha\beta} \\ \gamma_{\alpha\beta}^{\underline{m}} & 0 \end{bmatrix}. \tag{A.11}$$

In the case of the type IIB superstring the two Majorana-Weyl spinors have the same chirality, thus they transform in the same $SO(9,1)$ representation. Following [8] it is possible to construct explicitly the γ matrices from the $SO(8)$ gamma matrices which themselves are direct product of Pauli matrices:

$$\gamma_{\alpha\beta}^{i} = \begin{bmatrix} 0 & \sigma^{i\,a\dot{a}} \\ \sigma_{b\dot{b}}^{i} & 0 \end{bmatrix}, \tag{A.12}$$

where $i = 1, \ldots, 8$ and the $\sigma_{b\dot{b}}^{i}$ are the antisymmetric real $SO(8)$ Pauli matrices and they satisfy the following algebra:

$$\sigma_{a\dot{a}}^{i}\sigma_{\dot{a}b}^{j} + \sigma_{a\dot{a}}^{j}\sigma_{\dot{a}b}^{i} = 2\delta^{ij}\delta_{ab} \tag{A.13}$$

[6] Since we are describing gamma matrices in a flat space we adopt the standard notation for the indices: $m = 0, \ldots, 9$ is the $SO(9,1)$ vector index.

with $\sigma^i_{\dot{a}b}$ the transpose of $\sigma^i_{b\dot{a}}$.[7] A ninth one that anti-commutes with these eight is given by [11]:

$$\gamma^9_{\alpha\beta} = \gamma^{9\,\alpha\beta} = \begin{bmatrix} 1_8 & 0 \\ 0 & -1_8 \end{bmatrix}, \tag{A.15}$$

and the values of $\gamma^{0\,\alpha\beta}$ and $\gamma^0_{\alpha\beta}$ are similarly defined in order to be consistent with their algebra:

$$\gamma^0_{\alpha\beta} = \begin{bmatrix} -1_8 & 0 \\ 0 & -1_8 \end{bmatrix}, \qquad \gamma^{0\,\alpha\beta} = \begin{bmatrix} 1_8 & 0 \\ 0 & 1_8 \end{bmatrix}. \tag{A.16}$$

B. OPE

In this section the OPEs of the matter currents will be treated explicitly. The overall factor α^2 is omitted in the final results.

$$\begin{aligned} &J^{\underline{a}}(y)J^{\underline{d}}(z) = \alpha^2 < \partial X^{\underline{a}}(y)\partial X^{\underline{d}}(z) > + \ldots = \\ &= -\frac{1}{2\pi}\eta^{\underline{ad}}\partial_y\partial_z \log|y-z|^2 + \frac{1}{16\pi^2}\eta^{[\underline{ad}][\underline{ef}]}\int d^2\omega\{-\partial_y\overline{\partial}_{\overline{\omega}}\log|y-\omega|^2 N_{\underline{ef}}(\omega)\partial_z\log|\omega-z|^2 + \\ &+\partial_y\log|y-\omega|^2 N_{\underline{ef}}(\omega)\partial_z\overline{\partial}_{\overline{\omega}}\log|\omega-z|^2\} + \\ &+\frac{1}{16\pi^2}\eta^{[\underline{ad}][\underline{ef}]}\int d^2\omega\{-\partial_y\partial_\omega\log|y-\omega|^2\hat{N}_{\underline{ef}}(\omega)\partial_z\log|\omega-z|^2 + \\ &+\partial_y\log|y-\omega|^2\hat{N}_{\underline{ef}}(\omega)\partial_z\partial_\omega\log|\omega-z|^2\} + \mathcal{O}(J^2). \end{aligned} \tag{B.1}$$

The integrand containing $N_{\underline{ef}}$ is a δ function (up to a minus sign) and so it can be easily integrated. Furthermore all the currents are expanded around z, i.e. $\hat{N}_{\underline{ef}}(\omega) \cong \hat{N}_{\underline{ef}}(z) + \ldots$ and $N_{\underline{ef}}(\omega) \cong N_{\underline{ef}}(z) + \ldots$, the terms with the derivatives of the currents can be neglected at this order, just by dimensional analysis. Setting $z = 0$ the OPE becomes:

$$J^{\underline{a}}(y)J^{\underline{d}}(0) \simeq -\frac{\eta^{\underline{ad}}}{2\pi}\frac{1}{y^2} - \frac{1}{4\pi y}\eta^{[\underline{ad}][\underline{ef}]}N_{\underline{ef}}(0) + \frac{1}{4\pi}\eta^{[\underline{ad}][\underline{ef}]}\hat{N}_{\underline{ef}}(0)\frac{\overline{y}}{y^2}. \tag{B.2}$$

[7] A specific set for the σ matrices is given in [8]:

$$\begin{array}{llll} \sigma^1 = \epsilon\times\epsilon\times\epsilon & \sigma^2 = 1\times\tau_1\times\epsilon & \sigma^3 = 1\times\tau_3\times\epsilon & \sigma^4 = \tau_1\times\epsilon\times 1 \\ \sigma^5 = \tau_3\times\epsilon\times 1 & \sigma^6 = \epsilon\times 1\times\tau_1 & \sigma^7 = \epsilon\times 1\times\tau_3 & \sigma^8 = 1\times 1\times 1, \end{array} \tag{A.14}$$

where τ_i are the Pauli matrices and $\epsilon = \imath\tau_2$.

$$
\begin{aligned}
&J^{\underline{a}}(y)\overline{J}^{\underline{d}}(z) = (cl.) + \alpha^2 < \partial X^{\underline{a}}(y)\overline{\partial} X^{\underline{d}}(z) > + \ldots = \\
&= -\frac{1}{2\pi}\eta^{\underline{ad}}\partial_y\overline{\partial}_{\overline{z}}\log|y-z|^2 + \frac{1}{16\pi^2}\eta^{[\underline{ad}][\underline{ef}]}\int d^2\omega\{-\partial_y\overline{\partial}_{\overline{\omega}}\log|y-\omega|^2 N_{\underline{ef}}(\omega)\overline{\partial}_{\overline{z}}\log|\omega-z|^2 + \\
&+\partial_y\log|y-\omega|^2 N_{\underline{ef}}(\omega)\overline{\partial}_{\overline{z}}\overline{\partial}_{\overline{\omega}}\log|\omega-z|^2\} + \\
&+\frac{1}{16\pi^2}\eta^{[\underline{ad}][\underline{ef}]}\int d^2\omega\{-\partial_y\partial_\omega\log|y-\omega|^2\hat{N}_{\underline{ef}}(\omega)\overline{\partial}_{\overline{z}}\log|\omega-z|^2 + \\
&+\partial_y\log|y-\omega|^2\hat{N}_{\underline{ef}}(\omega)\overline{\partial}_{\overline{z}}\partial_\omega\log|\omega-z|^2\} \\
&\simeq -\frac{1}{4\pi(\overline{y}-\overline{z})}\eta^{[\underline{ad}][\underline{ef}]}N_{\underline{ef}}(z) - \frac{1}{4\pi(y-z)}\eta^{[\underline{ad}][\underline{ef}]}\hat{N}_{\underline{ef}}(z),
\end{aligned} \tag{B.3}
$$

where the first term in the second line is a δ function and it will be not considered here, since only singular terms are taken in account; the result (4.3) is obtained with $z = 0$. Since the procedure is completely analogous for the remaining bosonic components of the currents, we will not rewrite them, the results are listed in (4.4), (4.5).

In the case of the OPE between J_2J_1, J_2J_3, J_1J_1 and J_3J_3 there are contributions from the commutators in (4.1), since the X fields can propagate "freely", as one can understand from the entries of the matrix (3.1) and from the underlying super-algebra. We present explicitly only the OPE for the JJ components in each case, since for the other components the OPEs are completely analogous.

$$
\begin{aligned}
J^{\underline{a}}(y)J^{\delta}(0) &= \alpha^2(< \partial X^{\underline{a}}(y)\partial X^{\delta}(0) > + < \partial X^{\underline{a}}(y)[\widetilde{J}_3, X_2]^{\delta}(0) > + \\
&+ < [\widetilde{J}_3, X_3]^{\underline{a}}(y)\partial X^{\delta}(0) > + \ldots)
\end{aligned} \tag{B.4}
$$

The first term is

$$
\begin{aligned}
&< \partial X^{\underline{a}}(y)\partial X^{\delta}(0) >= \partial_y\partial_z < X^{\underline{a}}(y)X^{\delta}(z) > |_{z=0} = \\
&= \frac{1}{8\pi^2}f^{\underline{a}}_{\hat{\gamma}\hat{\rho}}\eta^{\hat{\gamma}\delta}\int d^2\omega\{-\partial_y\partial_\omega\log|y-\omega|^2\overline{J}^{\hat{\rho}}(\omega)\partial_z\log|\omega-z|^2 + \\
&+\partial_y\log|y-\omega|^2\overline{J}^{\hat{\rho}}\partial_z\partial_\omega\log|\omega-z|^2\}|_{z=0} = \frac{1}{2\pi}\frac{\overline{y}}{y^2}f^{\underline{a}}_{\hat{\gamma}\hat{\rho}}\eta^{\hat{\gamma}\delta}\overline{J}^{\hat{\rho}}(0),
\end{aligned} \tag{B.5}
$$

the second term:

$$
\begin{aligned}
< \partial X^{\underline{a}}(y)[J_3, X_2]^{\delta}(0) > &= - < \partial X^{\underline{a}}(y)f^{\delta}_{\underline{b}\hat{\rho}}X^{\underline{b}}(0)J^{\hat{\rho}}(0) >= \\
&= \frac{1}{2\pi}\eta^{\underline{ab}}\partial_y\log|y-z|^2 f^{\rho}_{\underline{b}\hat{\rho}}J^{\hat{\rho}}(z)|_{z=0} = \\
&= \frac{1}{2\pi y}f^{\underline{a}}_{\hat{\rho}\hat{\gamma}}\eta^{\hat{\gamma}\delta}J^{\hat{\rho}}(0),
\end{aligned} \tag{B.6}
$$

and the last term is:

$$
\begin{aligned}
< [J_3, X_3]^{\underline{a}}(y)\partial X^{\delta}(0) > &= < f^{\underline{a}}_{\hat{\rho}\hat{\gamma}}J^{\hat{\rho}}(y)X^{\hat{\gamma}}(y)\partial X^{\delta}(0) >= \\
&= -\frac{1}{2\pi}f^{\underline{a}}_{\hat{\rho}\hat{\gamma}}J^{\hat{\rho}}(y)\eta^{\hat{\gamma}\delta}\partial_z\log|y-z|^2|_{z=0} = \\
&= \frac{1}{2\pi y}f^{\underline{a}}_{\hat{\rho}\hat{\gamma}}J^{\hat{\rho}}(0)\eta^{\hat{\gamma}\delta}.
\end{aligned} \tag{B.7}
$$

Therefore one gets:

$$
J^{\underline{a}}(y)J^{\delta}(0) \simeq \frac{1}{2\pi}\frac{\overline{y}}{y^2}f^{\underline{a}}_{\hat{\gamma}\hat{\rho}}\eta^{\hat{\gamma}\delta}\overline{J}^{\hat{\rho}}(0) + \frac{1}{\pi y}f^{\underline{a}}_{\hat{\rho}\hat{\gamma}}\eta^{\hat{\gamma}\delta}J^{\hat{\rho}}(0). \tag{B.8}
$$

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$$J^{\underline{a}}(y)J^{\hat{\delta}}(0) \simeq \alpha^2(< \partial X^{\underline{a}}(y)\partial X^{\hat{\delta}}(0) > + < \partial X^{\underline{a}}(y)[\widetilde{J}_1, X_2]^{\hat{\delta}}(0) > + \\ < [\widetilde{J}_1, X_1]^{\underline{a}}(y)\partial X^{\hat{\delta}}(0) > + \ldots). \tag{B.9}$$

The first term:

$$< \partial X^{\underline{a}}(y)\partial X^{\hat{\delta}}(0) >= -\partial_y\partial_z(A^{-1}VA^{-1})^{\underline{a}\hat{\delta}}|_{z=0} = \\ = \frac{1}{8\pi^2} f^{\underline{a}}_{\rho\gamma}\eta^{\gamma\hat{\delta}} \int d^2\omega\{-\partial_y\overline{\partial}_{\overline{\omega}} \log|y-\omega|^2 J^\rho(\omega)\partial_z \log|\omega - z|^2 + \\ +\partial_y \log|y-\omega|^2 J^\rho(\omega)\overline{\partial}_{\overline{\omega}}\partial_z \log|\omega - z|^2\}|_{z=0} \tag{B.10}$$

The second term:

$$- < \partial X^{\underline{a}}(y) f^{\hat{\delta}}_{\underline{b}\rho} X^{\underline{b}}(0)J^\rho(0) > = \frac{1}{2\pi}\eta^{\underline{ab}} f^{\hat{\delta}}_{\underline{b}\rho} J^\rho(z)\partial_y \log|y-z|^2|_{z=0} = \\ = \frac{1}{2\pi} f^{\underline{a}}_{\rho\gamma}\eta^{\gamma\hat{\delta}} J^\rho(0)\frac{1}{y} \tag{B.11}$$

The third term:

$$< f^{\underline{a}}_{\rho\gamma} J^\rho(y)X^\delta(y)\partial X^{\hat{\delta}}(0) >= -\frac{1}{2\pi} f^{\underline{a}}_{\rho\gamma}\eta^{\gamma\hat{\delta}} J^\rho(y)\partial_z \log|y-z|^2|_{z=0} \tag{B.12}$$

Thus the OPE is:

$$J^{\underline{a}}(y)J^{\hat{\delta}}(0) \simeq -\frac{1}{2\pi y} f^{\underline{a}}_{\rho\gamma}\eta^{\gamma\hat{\delta}} J^\rho(0) + \frac{1}{2\pi y} f^{\underline{a}}_{\rho\gamma}\eta^{\gamma\hat{\delta}} J^\rho(0) + \frac{1}{2\pi y} f^{\underline{a}}_{\rho\gamma}\eta^{\gamma\hat{\delta}} J^\rho(0) \simeq \frac{1}{2\pi y} f^{\underline{a}}_{\rho\gamma}\eta^{\gamma\hat{\delta}} J^\rho(0). \tag{B.13}$$

$$J^\alpha(y)J^\delta(0) \simeq \alpha^2(< \partial X^\alpha(y)\partial X^\delta(0) > + < \partial X^\alpha(y)[J_2, X_3]^\delta(0) > + \\ < [J_2, X_3]^\alpha(y)\partial X^\delta(0) > + \ldots) \tag{B.14}$$

The first term:

$$< \partial X^\alpha(y)\partial X^\delta(0) >= -\partial_y\partial_z(A^{-1}VA^{-1})^{\alpha\delta}|_{z=0} = \\ = \frac{1}{8\pi^2} f^\alpha_{\underline{l}\hat{\gamma}}\eta^{\hat{\gamma}\delta} \int d^2\omega\{-\partial_y\partial_\omega \log|y-\omega|^2 \overline{J}^{\underline{l}}(\omega)\partial_z \log|\omega - z|^2 + \\ +\partial_y \log|y-\omega|^2 \overline{J}^{\underline{l}}(\omega)\partial_z\partial_\omega \log|\omega - z|^2\}|_{z=0} = \frac{1}{2\pi}\frac{\overline{y}}{y^2} f^\alpha_{\underline{l}\hat{\gamma}}\eta^{\hat{\gamma}\delta}\overline{J}^{\underline{l}}(0) \tag{B.15}$$

The second term:

$$< \partial X^\alpha(y) f^\delta_{\underline{l}\hat{\beta}} J^{\underline{l}}(0)X^{\hat{\beta}}(0) > = -\frac{1}{2\pi}\eta^{\alpha\hat{\beta}} f^\delta_{\underline{l}\hat{\beta}} J^{\underline{l}}(z)\partial_y \log|y-z|^2|_{z=0} \\ = \frac{1}{2\pi} f^\alpha_{\underline{l}\hat{\gamma}}\eta^{\hat{\gamma}\delta} J^{\underline{l}}(0)\frac{1}{y} \tag{B.16}$$

The third term:

$$< f^\alpha_{\underline{l}\hat{\beta}} J^{\underline{l}}(y)X^{\hat{\beta}}(y)\partial X^\delta(0) >= -\frac{1}{2\pi} f^\alpha_{\underline{l}\hat{\beta}} J^{\underline{l}}(y)\eta^{\hat{\beta}\delta}\partial_z \log|y-z|^2|_{z=0} \tag{B.17}$$

Therefore the OPE is given by:

$$J^{\alpha}(y)J^{\delta}(0) \simeq \frac{1}{2\pi}\frac{\overline{y}}{y^2} f^{\alpha}_{\underline{l}\hat{\gamma}}\eta^{\hat{\gamma}\delta}\overline{J}^{\underline{l}}(0) + \frac{1}{\pi y} f^{\alpha}_{\underline{l}\hat{\gamma}}\eta^{\hat{\gamma}\delta}J^{\underline{l}}(0). \tag{B.18}$$

$$\begin{aligned} J^{\hat{\alpha}}(y)J^{\hat{\delta}}(0) \simeq \alpha^2 (&< \partial X^{\hat{\alpha}}(y)\partial X^{\hat{\delta}}(0) > + < \partial X^{\hat{\alpha}}(y)[J_2, X_1]^{\hat{\delta}}(0) > + \\ &< [J_2, X_1]^{\hat{\alpha}}(y)\partial X^{\hat{\delta}}(0) > + \ldots) \end{aligned} \tag{B.19}$$

The first term is:

$$\begin{aligned} &< \partial X^{\hat{\alpha}}(y)\partial X^{\hat{\delta}}(0) >= -\partial_y\partial_z(A^{-1}VA^{-1})^{\hat{\alpha}\hat{\delta}}|_{z=0} = \\ &= \frac{1}{8\pi^2} f^{\hat{\alpha}}_{\underline{l}\gamma}\eta^{\gamma\hat{\delta}} \int d^2\omega\{-\partial_y\overline{\partial}_{\overline{\omega}} \log|y-\omega|^2 J^{\underline{l}}(\omega)\partial_z \log|\omega - z|^2 + \\ &+\partial_y \log|y-\omega|^2 J^{\underline{l}}(\omega)\partial_z\overline{\partial}_{\overline{\omega}} \log|\omega - z|^2\}|_{z=0} \end{aligned} \tag{B.20}$$

The second term is:

$$< \partial X^{\hat{\alpha}}(y) f^{\hat{\delta}}_{\underline{l}\beta} J^{\underline{l}}(0)X^{\beta}(0) >= \frac{1}{2\pi}\eta^{\hat{\alpha}\beta} f^{\hat{\delta}}_{\beta\underline{l}} J^{\underline{l}}(z)\partial_y \log|y-z|^2|_{z=0} = \frac{1}{2\pi y} f^{\hat{\alpha}}_{\underline{l}\gamma}\eta^{\gamma\hat{\delta}} J^{\underline{l}}(0) \tag{B.21}$$

The third term:

$$< [J_2, X_1]^{\hat{\alpha}}(y)\partial X^{\hat{\delta}}(0) >= -\frac{1}{2\pi} f^{\hat{\alpha}}_{\underline{l}\gamma}\eta^{\gamma\hat{\delta}} J^{\underline{l}}(y)\partial_z \log|y-z|^2|_{z=0} \tag{B.22}$$

Thus the OPE is:

$$\begin{aligned} J^{\hat{\alpha}}(y)J^{\hat{\delta}}(0) &\simeq -\frac{1}{2\pi y} f^{\hat{\alpha}}_{\underline{l}\gamma}\eta^{\gamma\hat{\delta}} J^{\underline{l}}(0)\frac{1}{2\pi y} f^{\hat{\alpha}}_{\underline{l}\gamma}\eta^{\gamma\hat{\delta}} J^{\underline{l}}(0) + \frac{1}{2\pi y} f^{\hat{\alpha}}_{\underline{l}\gamma}\eta^{\gamma\hat{\delta}} J^{\underline{l}}(0) \simeq \\ &\simeq \frac{1}{2\pi y} f^{\hat{\alpha}}_{\underline{l}\gamma}\eta^{\gamma\hat{\delta}} J^{\underline{l}}(0). \end{aligned} \tag{B.23}$$

In the case of $J_3 J_1$ OPE the same algebraic structure of $J_2 J_2$ is involved, therefore we present briefly the result.

$$\begin{aligned} &J^{\hat{\alpha}}(y)J^{\delta}(0) = (cl.) + \alpha^2 < \partial X^{\hat{\alpha}}(y)\partial X^{\delta}(0) > + \ldots \simeq \\ &\simeq (-\frac{1}{2\pi}\eta^{\hat{\alpha}\delta}\partial_z\partial_y \log|y-z|^2 + \\ &+\frac{1}{16\pi^2}\eta^{\hat{\alpha}\beta} f^{[ef]}_{\beta\hat{\gamma}}\eta^{\hat{\gamma}\delta} \int d^2\omega[-\partial_y\overline{\partial}_{\overline{\omega}} \log|y-\omega|^2 N_{\underline{ef}}(\omega)\partial_z \log|\omega - z|^2 + \\ &+\partial_y \log|y-\omega|^2 N_{\underline{ef}}(\omega)\partial_z\overline{\partial}_{\overline{\omega}} \log|\omega - z|^2 - \partial_y\partial_\omega \log|y-\omega|^2 \hat{N}_{\underline{ef}}(\omega)\partial_z \log|\omega - z|^2 + \\ &+\partial_y \log|y-\omega|^2 \hat{N}_{\underline{ef}}(\omega)\partial_z\partial_\omega \log|\omega - z|^2])|_{z=0} \simeq \\ &\simeq -\frac{1}{2\pi y^2}\eta^{\hat{\alpha}\delta} + \frac{1}{4\pi}\frac{\overline{y}}{y^2}\eta^{\hat{\alpha}\beta} f^{[ef]}_{\beta\hat{\gamma}}\eta^{\hat{\gamma}\delta}\hat{N}_{\underline{ef}}(0) - \frac{1}{4\pi y}\eta^{\hat{\alpha}\beta} f^{[ef]}_{\beta\hat{\gamma}}\eta^{\hat{\gamma}\delta} N_{\underline{ef}}(0), \end{aligned} \tag{B.24}$$

where the classical term is omitted.

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C. Classical equations of motion

We now derive the classical equations of motion from the action (2.1). Under small variations of the fields g the currents satisfy [5, 9]:

$$
\begin{array}{c}
\delta g = gX \qquad \delta g^{-1} = -Xg^{-1}, \qquad X \in H_i \\
\delta J_i = \partial X + [J, X]_i \qquad \overline{J}_i = \overline{\partial} X + [\overline{J}, X]_i \\
\delta J_0 = [J, X]_0 \qquad \delta \overline{J}_0 = [\overline{J}, X]_0 \\
\delta N = [N, \Lambda] \qquad \delta \hat{N} = [\hat{N}, \hat{\Lambda}],
\end{array}
\tag{C.1}
$$

where for the variation of the Lorentz ghost currents the gauge transformation is used since this is the most general covariant variation which respects the $SO(4,1) \times SO(5)$ symmetry [5]. Furthermore under gauge transformation for the pure spinor actions S_λ and $\hat{S}_{\hat{\lambda}}$ we have $\delta S_\lambda = -N^{[cd]} \partial \Lambda_{[cd]}$ and $\delta \hat{S}_{\hat{\lambda}} = -\hat{N} \overline{\partial} \hat{\Lambda}_{[cd]}$. Plugging (C.1) in the action (2.1) and using the Maurer-Cartan identities $\partial \overline{J} - \overline{\partial} J + [J, \overline{J}] = 0$ one gets:

$$
\begin{aligned}
\overline{\nabla} J_2 &= [J_3, \overline{J}_3] + \frac{1}{2}[N, \overline{J}_2] - \frac{1}{2}[J_2, \hat{N}] \\
\nabla \overline{J}_2 &= -[J_1, \overline{J}_1] + \frac{1}{2}[N, \overline{J}_2] - \frac{1}{2}[J_2, \hat{N}] \\
\overline{\nabla} J_3 &= \frac{1}{2}[N, \overline{J}_3] - \frac{1}{2}[J_3, \hat{N}] \\
\nabla \overline{J}_3 &= -[J_1, \overline{J}_2] - [J_2, \overline{J}_1] + \frac{1}{2}[N, \overline{J}_3] - \frac{1}{2}[J_3, \hat{N}] \\
\overline{\nabla} J_1 &= [J_3, \overline{J}_2] + [J_2, \overline{J}_3] + \frac{1}{2}[N, \overline{J}_1] - \frac{1}{2}[J_1, \hat{N}] \\
\nabla \overline{J}_1 &= \frac{1}{2}[N, \overline{J}_1] - \frac{1}{2}[J_1, \hat{N}] \\
\overline{\nabla} N &= \frac{1}{2}[N, \hat{N}] \\
\nabla \hat{N} &= -\frac{1}{2}[N, \hat{N}],
\end{aligned}
\tag{C.2}
$$

where the covariant derivatives are $\nabla = \partial + [J_0, \ ]$ and $\overline{\nabla} = \overline{\partial} + [\overline{J}_0, \ ]$.

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